

For each problem, write and solve a system of equations. **You must use one of the THREE approved methods.** Make sure you define your variables and answer in a complete sentence!

1. A rectangle has a perimeter of 14 feet. The length is equal to 1 less than 4 times its width. Find the dimensions of the rectangle.

$L = \# \text{ of feet for length}$   
 $W = \# \text{ of feet for width}$

$$\begin{cases} L = 4W - 1 & \textcircled{1} \\ 2L + 2W = 14 & \textcircled{2} \end{cases}$$

[substitution]

step 1:  $\checkmark$   
 step 2:  $\textcircled{2} \ 2(4W - 1) + 2W = 14$   
 $8W - 2 + 2W = 14$   
 $10W = 16$   
 $W = 1.6 \text{ ft}$

step 3:  $\textcircled{1} \ L = 4(1.6) - 1$   
 $L = 6.4 - 1$   
 $L = 5.4 \text{ ft}$

step 4:  $\textcircled{1} \ 5.4 = 4(1.6) - 1$   
 check  $5.4 = 6.4 - 1 \checkmark$   
 $\textcircled{2} \ 2(5.4) + 2(1.6) = 14$   
 $10.8 + 3.2 = 14 \checkmark$

The dimensions of the rectangle are 1.6 ft by 5.4 ft.

2. Two cars get an oil change at the same service center. Each customer is charged a fixed fee for the oil change plus a certain amount per quart of oil used. The oil change for a car that requires 5 quarts of oil costs \$22.45. The oil change for a car that requires 7 quarts of oil costs \$25.45. Find the fixed fee and the cost per quart of oil.

$F = \# \text{ of } \$ \text{ for the fee}$   
 $Q = \# \text{ of } \$ \text{ per quart of oil}$

$$\begin{cases} F + 5Q = 22.45 & \textcircled{1} \\ F + 7Q = 25.45 & \textcircled{2} \end{cases}$$

[combination]

step 1:  $\xrightarrow{-\textcircled{1}}$   $-F + (-5Q) = -22.45$   
 $\rightarrow F + 7Q = 25.45$

step 2:  $2Q = 3$   
 $Q = \$1.50$

step 3:  $\textcircled{1} \ F + 5(1.5) = 22.45$   
 $F + 7.5 = 22.45$   
 $F = \$14.95$

step 4:  $\textcircled{1} \ 14.95 + 5(1.5) = 22.45$   
 check  $14.95 + 7.5 = 22.45 \checkmark$   
 $\textcircled{2} \ 14.95 + 7(1.5) = 25.45 \checkmark$   
 $14.95 + 10.5 = 25.45 \checkmark$

The fixed fee is \$14.95 and they charge \$1.50 per quart of oil.

3. For a floral arrangement class, Alice has to create an arrangement of twigs and flowers that has a total of 9 objects. She has to pay for the twigs and flowers that she uses in her arrangement. Each twig costs \$1 and each flower costs \$3. If Alice spent \$15 on her arrangement, how many of each object did she use?

$$\begin{array}{l}
 T = \text{\# of twigs} \\
 F = \text{\# of flowers}
 \end{array}
 \left\{
 \begin{array}{l}
 T + F = 9 \quad \textcircled{1} \\
 T + 3F = 15 \quad \textcircled{2}
 \end{array}
 \right.
 \begin{array}{l}
 \text{[combination]} \\
 \text{step 2:} \\
 \xrightarrow{\text{step 2:}} \quad \xrightarrow{\text{step 2:}} \quad \xrightarrow{\text{step 2:}} \\
 \begin{array}{l}
 -T + (-F) = -9 \\
 \underline{T + 3F = 15} \\
 2F = 6 \\
 F = 3
 \end{array}
 \end{array}
 \begin{array}{l}
 \text{step 3:} \textcircled{1} \quad T + 3 = 9 \\
 T = 6 \\
 \text{step 4:} \textcircled{1} \quad 6 + 3 = 9 \checkmark \\
 \text{check} \textcircled{2} \quad 6 + 3(3) = 15 \\
 6 + 9 = 15 \checkmark
 \end{array}$$

Alice used 6 twigs and 3 flowers in her arrangement.

4. Thomas and Patrick each downloaded some songs on Saturday. The site they use charges the same price for each "regular song" and a different price for each "new release". Thomas bought 3 regular songs and 2 new releases for \$12.85. Patrick bought 1 regular song and 2 new releases for \$8.95. Determine what the site charges for each type of song.

$$\begin{array}{l}
 R = \text{\# of \$ for 1 regular song} \\
 N = \text{\# of \$ for 1 new song}
 \end{array}
 \left\{
 \begin{array}{l}
 3R + 2N = 12.85 \quad \textcircled{1} \\
 R + 2N = 8.95 \quad \textcircled{2}
 \end{array}
 \right.
 \begin{array}{l}
 \text{step 2:} \\
 \xrightarrow{\text{step 2:}} \quad \xrightarrow{\text{step 2:}} \quad \xrightarrow{\text{step 2:}} \\
 \begin{array}{l}
 3R + 2N = 12.85 \\
 \underline{-R + (-2N) = -8.95} \\
 2R = 3.90 \\
 R = \$1.95
 \end{array}
 \end{array}
 \begin{array}{l}
 \text{step 3:} \textcircled{2} \quad 1.95 + 2N = 8.95 \\
 2N = 7 \\
 N = \$3.50 \\
 \text{step 4:} \textcircled{1} \quad 3(1.95) + 2(3.5) = 12.85 \\
 \text{check} \quad 5.85 + 7 = 12.85 \checkmark \\
 \textcircled{2} \quad 1.95 + 2(3.5) = 8.95 \\
 1.95 + 7 = 8.95 \checkmark
 \end{array}$$

Each regular song costs \$1.95 and each new release costs \$3.50.

5. A sports equipment store is having a sale on soccer balls. A soccer coach purchases 10 soccer balls and 2 soccer ball bags for \$155. Another soccer coach purchases 12 soccer balls and 3 soccer ball bags for \$189. Find the cost of a soccer ball and a soccer ball bag.

$$\begin{array}{l}
 B = \# \text{ of } \$ \text{ for 1 ball} \\
 G = \# \text{ of } \$ \text{ for 1 bag}
 \end{array}
 \left\{
 \begin{array}{l}
 10B + 2G = 155 \text{ ①} \\
 12B + 3G = 189 \text{ ②}
 \end{array}
 \right.
 \begin{array}{l}
 \text{[Combination]} \\
 \text{step 1: } \times (-3) \rightarrow -30B + (-6G) = -465 \\
 \text{step 2: } \times 2 \rightarrow 24B + 6G = 378 \\
 \text{step 3: } \text{① } 10(14.5) + 2G = 155 \\
 \qquad \qquad \qquad 145 + 2G = 155 \\
 \qquad \qquad \qquad 2G = 10 \\
 \qquad \qquad \qquad G = 5 \\
 \text{step 4: } \text{① } -6B = -87 \\
 \qquad \qquad \qquad B = 14.50
 \end{array}$$

step 4: ①  $10(14.5) + 2(5) = 155$   
 check  $145 + 10 = 155 \checkmark$   
 ②  $12(14.5) + 3(5) = 189$   
 $174 + 15 = 189 \checkmark$

Each soccer ball costs \$14.50 and each soccer ball bag costs \$5.

6. You are planning a birthday party for your 8 year old cousin. You can have a party at a pizza place for \$8 per person plus a \$30 "party fee" for favors and clean-up. A taco place has a similar deal but it costs \$12 per person plus a \$14 "party fee". How many children would you have to invite for the party to cost the same at both places? What would this party cost?

$$\begin{array}{l}
 C = \# \text{ of children} \\
 T = \# \text{ of } \$ \text{ in the total cost}
 \end{array}
 \left\{
 \begin{array}{l}
 T = 8C + 30 \text{ ①} \\
 T = 12C + 14 \text{ ②}
 \end{array}
 \right.
 \begin{array}{l}
 \text{[Substitution]} \\
 \text{step 1: } \text{① } \checkmark \\
 \text{step 2: } \text{② } 8C + 30 = 12C + 14 \\
 \qquad \qquad \qquad -4C = -16 \\
 \qquad \qquad \qquad C = 4
 \end{array}$$

step 3: ①  $T = 8(4) + 30$   
 $T = 32 + 30$   
 $T = 62$

step 4: ①  $62 = 8(4) + 30$   
 check  $62 = 32 + 30 \checkmark$

②  $62 = 12(4) + 14$   
 $62 = 48 + 14 \checkmark$

The party will be \$62 at both places if you invite 4 children.

7. A test has only 2-point and 5-point questions. It is worth 70 points and has 23 questions. How many of each type of question are on the test?

$$\begin{array}{l}
 T = \# \text{ of 2-point questions} \\
 F = \# \text{ of 5-point questions}
 \end{array}
 \left\{
 \begin{array}{l}
 T + F = 23 \text{ ①} \\
 2T + 5F = 70 \text{ ②}
 \end{array}
 \right.
 \begin{array}{l}
 \text{Step 1:} \\
 \times (-2) \rightarrow -2T + (-2F) = -46 \\
 \hline
 \rightarrow 2T + 5F = 70 \\
 \hline
 \text{Step 2: } 3F = 24 \\
 F = 8 \text{ questions} \\
 \text{Step 3: ① } T + 8 = 23 \\
 T = 15 \text{ questions} \\
 \text{Step 4: ① } 15 + 8 = 23 \checkmark \\
 \text{check ② } 2(15) + 5(8) = 70 \\
 30 + 40 = 70 \checkmark
 \end{array}$$

There are 15 2-pt questions and 8 5-pt questions on the test.

8. Your toilets clog up and you have to hire a plumber to come and ream out your pipes. The first plumber you call only charges \$35 to walk in, but charges \$50 an hour. The second plumber you call only charges \$40 an hour, but she charges \$60 to walk in. When is it better to use each plumber?

$$\begin{array}{l}
 H = \# \text{ of hours} \\
 T = \# \text{ of } \$ \text{ for Total charge}
 \end{array}
 \left\{
 \begin{array}{l}
 T = 50H + 35 \text{ ①} \\
 T = 40H + 60 \text{ ②}
 \end{array}
 \right.
 \begin{array}{l}
 \text{Step 1: ① } \checkmark \\
 \text{Step 2: ② } 50H + 35 = 40H + 60 \\
 10H = 25 \\
 H = 2.5 \text{ hours} \\
 \text{Step 3: ① } T = 50(2.5) + 35 \\
 T = 125 + 35 \\
 T = \$160 \\
 \text{Step 4: ① } 160 = 50(2.5) + 35 \\
 \text{check } 160 = 125 + 35 \checkmark \\
 \text{② } 160 = 40(2.5) + 60 \\
 160 = 100 + 60 \checkmark
 \end{array}$$

Both plumbers would charge \$160 for  $2\frac{1}{2}$  hours of work.

It will be cheaper to use Plumber #1 for small jobs (less than  $2\frac{1}{2}$  hrs).  
 It will be cheaper to use Plumber #2 for larger jobs (over  $2\frac{1}{2}$  hrs).

9. A website allows users to download individual songs or an entire album. All individual songs cost the same to download, and all albums cost the same to download. Ryan paid \$14.94 to download 5 individual songs and 1 album. Seth paid \$22.95 to download 3 individual songs and 2 albums. How much does the website charge to download an individual song and how much does it charge for an entire album?

$S$  = # of \$ for  
1 song

$A$  = # of \$ for  
1 album

$$\begin{cases} 5S + A = 14.94 & \textcircled{1} \\ 3S + 2A = 22.95 & \textcircled{2} \end{cases}$$

{Substitution}

Step 1:  $\textcircled{1} A = -5S + 14.94$

Step 2:  $\textcircled{2} 3S + 2(-5S + 14.94) = 22.95$

$$3S + (-10S) + 29.88 = 22.95$$

$$-7S = -6.93$$

$$S = \$9.99$$

Step 3:  $\textcircled{1} A = -5(9.99) + 14.94$

$$A = -4.95 + 14.94$$

$$A = \$9.99$$

Step 4:  $\textcircled{1} 5(9.99) + 9.99 = 14.94$

check  $4.95 + 9.99 = 14.94 \checkmark$

$\textcircled{2} 3(9.99) + 2(9.99) = 22.95$

$$2.97 + 19.98 = 22.95 \checkmark$$

Each song costs \$9.99 and  
each album costs \$9.99.

10. A new on-line movie rental company is advertising two different plans for "vintage" pay-per-view movies (movies that came out more than 10 years ago).

Plan A: You pay \$2 per movie.

Plan B: You pay a yearly membership fee of \$18 and movies are discounted to \$1.50 per movie.

Which plan would you choose? You must provide a mathematical explanation for your choice.

$M$  = # of movies

$T$  = # of \$ for the  
total cost

$$\begin{cases} T = 2M & \textcircled{1} \\ T = 1.5M + 18 & \textcircled{2} \end{cases}$$

{Substitution}

Step 1:  $\textcircled{1} \checkmark$

Step 2:  $\textcircled{2} 2M = 1.5M + 18$

$$.5M = 18$$

$$M = 36 \text{ movies}$$

Step 3:  $\textcircled{1} T = 2(36)$

$$T = \$72$$

Step 4:  $\textcircled{1} 72 = 2(36) \checkmark$

check

$\textcircled{2} 72 = 1.5(36) + 18$

$$72 = 54 + 18 \checkmark$$

Both plans would cost \$72  
if you rent exactly 36 movies.

If you would rent less than  
36 movies, Plan A is cheaper.

If you would rent more than  
36 movies, Plan B is cheaper.

Since I don't rent a lot of  
movies (less than 36 per year),

I would choose Plan A.

11. Matt invested \$2000 in stocks and bonds. This year the bonds paid 8% interest, and the stocks paid 6% in dividends. Matt received a total of \$144 in interest and dividends. How much money did he put in each type of investment?

$S$  = # of \$ in stocks

$B$  = # of \$ in bonds

$$\begin{cases} S + B = 2000 & \textcircled{1} \\ .06S + .08B = 144 & \textcircled{2} \end{cases}$$

[Substitution]

Step 1:  $\textcircled{1} S = -B + 2000$

Step 2:  $\textcircled{2} .06(-B + 2000) + .08B = 144$   
 $-.06B + 120 + .08B = 144$   
 $.02B = 24$   
 $B = \$1200$

Step 3:  $\textcircled{1} S = -1200 + 2000$   
 $S = \$800$

Step 4:  $\textcircled{1} 800 + 1200 = 2000 \checkmark$

Check  $\textcircled{2} .06(800) + .08(1200) = 144$   
 $48 + 96 = 144 \checkmark$

Matt invested \$800 in stocks and \$1200 in bonds.

12. At a grocery store, a customer paid a total of \$9.70 for 1.8 pounds of potato salad and 1.4 pounds of coleslaw. Another customer paid a total of \$6.55 for 1 pound of potato salad and 1.2 pounds of coleslaw. How much does the grocery store charge for 1 pound of potato salad and how much does the grocery store charge for 1 pound of coleslaw?

$P$  = # of \$ per pound of potato salad

$C$  = # of \$ per pound of coleslaw

$$\begin{cases} 1.8P + 1.4C = 9.70 & \textcircled{1} \\ P + 1.2C = 6.55 & \textcircled{2} \end{cases}$$

[Substitution]

Step 1:  $\textcircled{2} P = -1.2C + 6.55$

Step 2:  $\textcircled{1} 1.8(-1.2C + 6.55) + 1.4C = 9.70$   
 $-2.16C + 11.79 + 1.4C = 9.70$   
 $-.76C = -2.09$   
 $C = \$2.75$

Step 3:  $\textcircled{2} P = -1.2(2.75) + 6.55$   
 $P = \$3.25$

Step 4:  $\textcircled{1} 1.8(3.25) + 1.4(2.75) = 9.70$   
 Check  $5.85 + 3.85 = 9.70 \checkmark$

$\textcircled{2} 3.25 + 1.2(2.75) = 6.55$   
 $3.25 + 3.30 = 6.55 \checkmark$

Each pound of potato salad costs \$3.25 and each pound of coleslaw costs \$2.75.